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EXPERIMENTAL INVESTIGATION OF THE TEMPERATURE DISTRIBUTION IN A HORIZONTAL LAYER OF FLUID HEATED FROM BELOW

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Abstract—An experimental investigation was made of the temperature distribution and fluid flow in a horizontal layer of liquid heated from below. The experiments covered a range of Rayleigh numbers from $1{\cdot}16 \times 10^3$ to $2{\cdot}03 \times 10^6$. The upper surface of the liquid was, in some cases, open to the surroundings and, in other cases, in contact with a glass plate. Silicone fluids of viscosity 2, 50 and 1000 cSt were used. The flow patterns were observed by mixing a small quantity of aluminum dust with the fluids.

The fluid flow in the laminar regime consisted of a combination of "rolls" and square planform cells. The measurements of temperature did not agree closely with theoretical predictions.

In the transition regime the flow pattern was cellular. The temperature measurements were compared with theory and a considerable discrepancy was found.

For high Rayleigh numbers the time mean temperature distribution close to the boundaries was inversely proportional to the cube root of the distance from the boundary. In one case (a high Prandtl-number fluid) there was an additional region close to the surface where the temperature was inversely proportional to the distance from the surface. The agreement with theory was quite good. The nature of the heat transport processes was also studied. The level of the r.m.s. temperature fluctuations was much less than the theoretical values.

NOMENCLATURE

- A, constant of proportionality in equation (2) [dimensionless];
- C, specific heat of experimental fluid (see definition of Rayleigh number) [Btu/lb m degF];
- g, acceleration of gravity [$4{\cdot}170 \times 10^8$ ft/h^2];
- h, coefficient of heat transfer [Btu/h ft^2 degF];
- k, thermal conductivity [Btu/h ft^2 degF/ft];
- L, distance between upper and lower surfaces of fluid [ft];
- n, exponent in equation (2) [dimensionless];
- Nu, Nusselt number (hL/k) [dimensionless];
- R, ratio of convective to total heat transport [dimensionless];
- Ra, Rayleigh number ($g\beta CL^3\Delta T/\nu k$) [dimensionless];
- T, temperature [°F];
- $\bar{T}$, time and space average temperature [°F];
- T_c, temperature of upper (cold) surface of fluid layer [°F];
- T_H, temperature of lower (hot) surface of fluid layer [°F];
- T_M, mean temperature of fluid layer [$(T_H + T_c)/2$] [°F];
- x, coordinate in horizontal plane [ft];
- x', dimensionless coordinate (x/L);
- y, coordinate in horizontal plane [ft];
- y', dimensionless coordinate (y/L);
- z, coordinate in vertical direction [ft];
- z', dimensionless coordinate in vertical direction (z/L);

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z_α, value of vertical coordinate (z) at which Peclet number reaches critical value (reference [12]) [ft];
z_v, value of vertical coordinate (z) at which Reynolds number reaches critical value (reference [12]) [ft];
ΔT, temperature difference between upper and lower surface ($T_H - T_c$) [degF];
β, coefficient of volume expansion [degF^{-1}];
θ, dimensionless temperature ($T - T_M/T_H - T_c$);
$\bar{\theta}$, dimensionless time and space average temperature ($\bar{T} - T_M/T_H - T_c$);
$\tilde{\theta}'$, root mean square value of the temperature fluctuations [degF];
$\overline{\overline{\theta'^2}}$, mean square value of the temperature fluctuations averaged over all space [degF^2];
ν, kinematic viscosity [ft^2/h];
ξ, Howard's dimensionless height (0·464 $Ra^{\frac{1}{3}} z'$);
ρ, density [lb/ft^3].

INTRODUCTION

THE PROBLEM of convective motion and heat transfer through a layer of fluid heated from below has been the subject of a number of theoretical [3, 10, 12, 14, 17–20, 25] and experimental [4–7, 9, 13, 16, 21–23, 26–28] investigations.

Fluid motion does not commence on the first application of heat to such a system but must await the attainment of a certain stability condition, expressed by the critical Rayleigh number (Ra). The flow which initially appears has a regular flow pattern (see Fig. 1). If the Rayleigh number is increased it is observed that the flow pattern gradually breaks up (Fig. 2) until it becomes completely disordered (Figs. 3 and 4). This second transition in the fluid motion from "laminar" to "turbulent" flow* is accompanied by a change in the nature of the heat transfer [23].

Most of the experimental investigations of free convection in fluid layers have either considered the overall effect of the fluid properties and configuration on the heat transfer or have been studies of the flow pattern; there is a lack of investigations into the details of the fluid flow. In the laminar regime some temperature distributions in a thin liquid layer have been reported [28]. For turbulent flow, Thomas and Townsend [26] have measured the mean temperature profiles and distribution of temperature fluctuations in a layer of air confined between two horizontal, rigid surfaces. These measurements did not fully support the theoretical temperature profiles of Malkus [14] and appear to have been considered by the authors to be rather inconclusive.

Townsend [27], Thomas and Townsend [26] and Croft [4] have investigated the mean temperature distribution and temperature fluctuations in air above a heated plate. It is possible that such a system might correspond to the lower portion of a layer of air which is extremely thick or in which the Rayleigh number is very high. These measurements did not agree completely with the theories of Malkus [14] or Priestley [19], the latter being specifically for the case of a body of fluid of unrestricted depth.

The mixing-length theory was used by Kraichnan [12] to derive expressions for the mean temperature profile and the temperature fluctuations in a fluid layer in turbulent motion. In 1963 Howard [11] published a theoretical relation between the temperature and the height above the heated surface for a confined fluid layer. Howard compared his results with Townsend's data and found reasonable agreement, however, such a procedure is not completely satisfactory because the experiments apply to free convection into a semi-infinite region.

The investigations reported here were intended to measure the temperature distribution

* It is possible that the disordered motion observed in the experiments is not truly turbulent (see discussion of reference [7]) but in the absence of any other suitable descriptive term it will be so called in this paper.

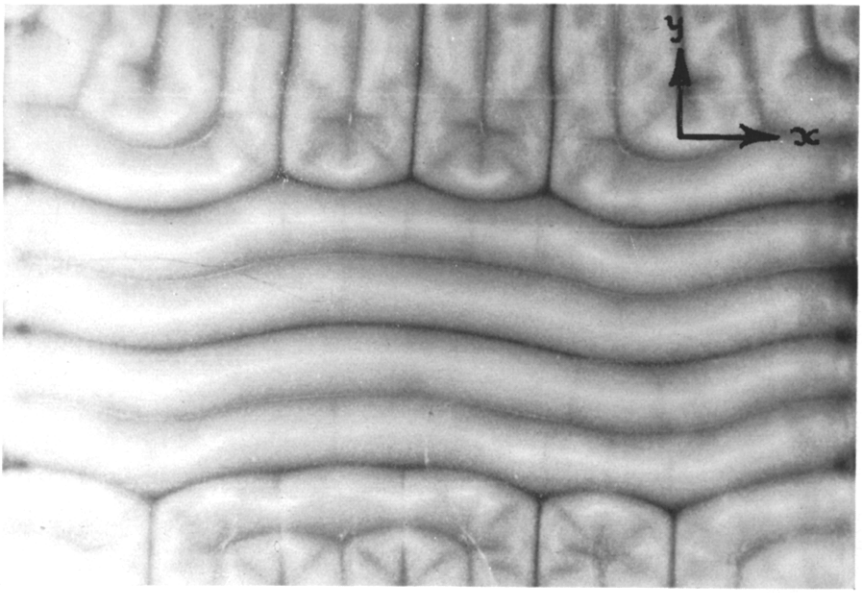

FIG. 1. Flow visualization, rigid–rigid system, $Ra = 3{\cdot}04 \times 10^3$.

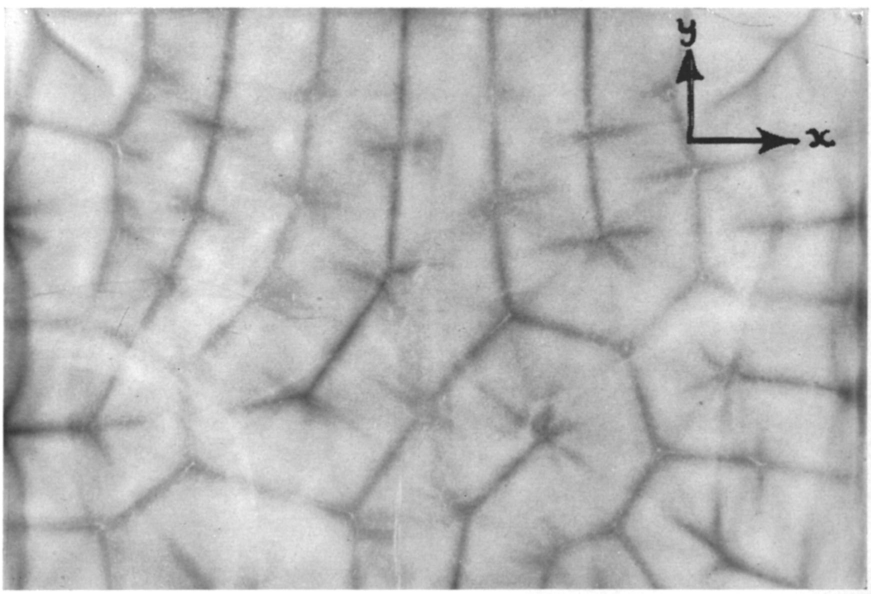

FIG. 2. Flow visualization, rigid–rigid system, $Ra = 2{\cdot}58 \times 10^4$.

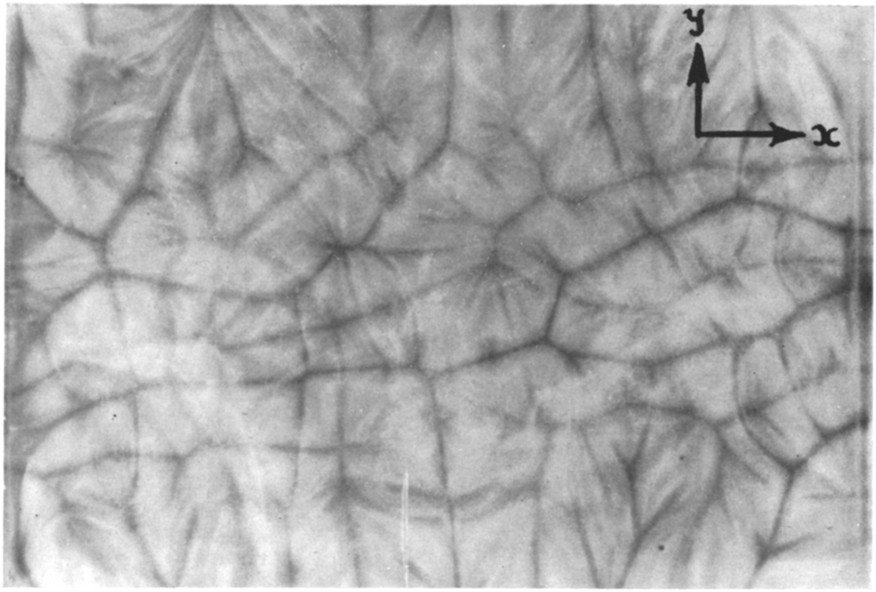

FIG. 3. Flow visualization, rigid–rigid system, $Ra = 1{\cdot}31 \times 10^5$.

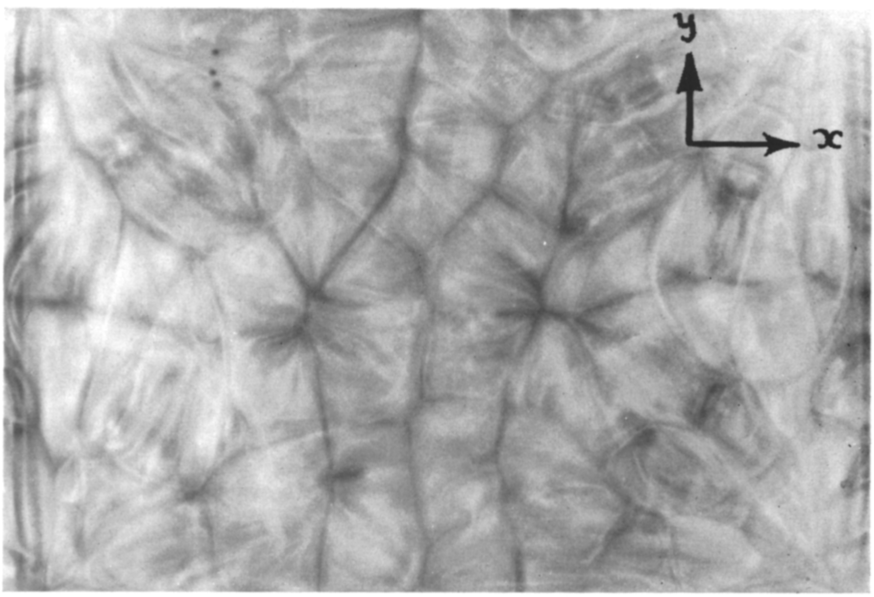

FIG. 4. Flow visualization, rigid–rigid system, $Ra = 5{\cdot}45 \times 10^4$.

in a liquid layer heated from below. The upper surface of the liquid was, in some cases, open to the surroundings (hereafter this will be called the rigid–free problem) and in other cases, in contact with a glass plate (to be called the rigid–rigid problem).

APPARATUS

The heater plate was made of copper $9\frac{3}{4}$-in by $7\frac{3}{4}$-in by $\frac{1}{2}$-in thick and attached by screws was a copper compression plate, $\frac{1}{4}$-in thick. Thus, the heater plate was $\frac{3}{4}$-in thick and sandwiched between its two sections was an electrical heating element. The upper side of the plate which was in contact with the experimental fluid was machined smooth and carefully polished to a mirror finish. The heating element consisted of a sheet of 0·008-in thick nichrome V-cut into parallel strips and joined at their ends so as to form one continuous current carrying strip [7]. The element was electrically insulated from the copper plates by being sandwiched between two 0·01-in thick sheets of "Micanite".

D.C. power was supplied to the heater to minimize the effect of electrical noise on the temperature measuring system. The rectifier was connected across a "Sola" constant voltage transformer to guard against variations in the line voltage. Measurements of voltage and amperage indicated the power input to the heater plate.

The walls of the test chamber were formed by cutting a rectangular hole 8 in by 6 in in a $\frac{7}{16}$-in thick piece of "Micarta" having overall dimensions of $17\frac{3}{4}$ in by $15\frac{3}{4}$ in (see Fig. 5).

In experiments where an upper surface was required, a $\frac{1}{4}$-in thick glass plate was used. In this way, the flow pattern inside the chamber could be viewed by mixing a very small quantity (0·015 per cent of the fluid weight) of aluminum dust with the experimental fluid.*

* It was found that the particles would not remain in suspension in the 2-cSt silicone fluid, so it was only possible to examine the flow patterns in the higher viscosity fluids.

The upper surface of the fluid was cooled either by the free convection (rigid–free and rigid–rigid boundary conditions) or by a small blower (rigid–rigid case only) which directed a stream of air normal to the glass plate.

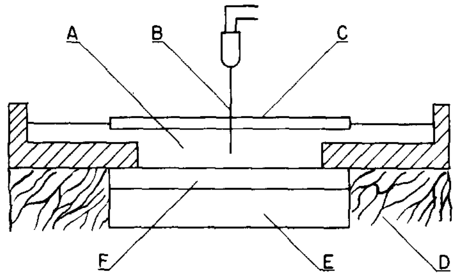

FIG. 5. Schematic diagram of test chamber.
A. Experimental fluid.
B. Thermocouple probe.
C. Movable glass plate (ball bearings not shown for clarity).
D. Glass wool.
E. Asbestos.
F. Heater plate.

To measure the temperatures in the chamber when the glass plate was in place, a calibrated probe, shown in Fig. 6, was inserted through a 0·04-in diameter hole at the center of the plate.

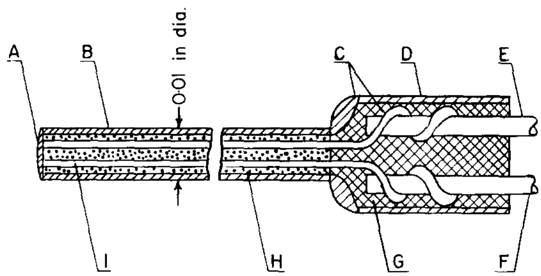

FIG. 6. Probe for use with rigid upper surface (not to scale).
A. Weld.
B. Inconel sheath (0·002-in wall thickness).
C. Brazing.
D. Ferrule.
E. Alumel wire.
F. Chromel wire.
G. Potting compound.
H. Packed magnesium oxide powder.
I. Chromel and alumel thermocouple wires (0·0015-in diameter).

The probe was moved vertically by a micrometer screwhead, which was capable of measuring to 0·001 in. The glass plate was used to support the probe, so that when it was moved horizontally the probe also moved. The position of the probe in the horizontal plane was indicated by a pointer rigidly connected to the probe support frame and moving over two mutually perpendicular scales placed alongside the apparatus. For ease of movement, the glass plate was carried on ball bearings and moved by driving screws located in the side walls.

Where a free upper surface was used, the temperature of the fluid at various points was measured by a calibrated probe designed for use in connection with another investigation [15]. Figure 7 indicates the form of this probe.

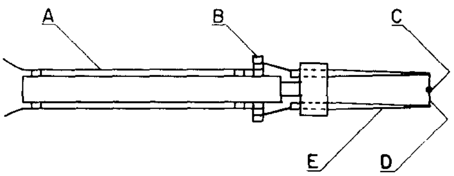

FIG. 7. Probe for use with free upper surface (not to scale).
A. Lead wires epoxy cemented to stem.
B. Terminal lugs.
C. Welded junction.
D. Chromel and alumel thermocouple wires (0·001-in diameter).
E. Pyrex glass tube (0·016-in minimum diameter 0·125-in maximum diameter).

The probe support was connected to a driving screw with fine pitch having the minimum of backlash. The location of the probe was determined by a dial gage connected to the driving screw. The drive mechanism and dial gage were carried on a bridge provided with levelling screws.

The e.m.f. output of the probe thermocouple was recorded on an Offner Dynograph recording oscillograph, which was able to measure to the nearest microvolt (0·1 degF).

To minimize the effects of electrical noise, the thermocouple circuit was completely enclosed in grounded shielding. In this way, stray electromagnetic effects were kept to a level below that corresponding to a 1-mV reading on the recorder.

The lower temperature was measured by copper–constantan thermocouples embedded in the heater plate $\frac{1}{64}$ in below the liquid–metal interface. The temperature of the glass–liquid interface was determined by two copper–constantan thermocouples. The lead wires (36 gage) were laid on the upper surface of the glass and led through two small holes to a shallow groove cut in the underside of the glass, where the junction was located.

The test chamber was mounted on a support with three levelling screws, which were placed on rubber vibration isolators. The apparatus was surrounded by the insulation shown in Fig. 5 and by a one-foot-high draft screen.

The fluids employed were Dow Corning silicone oils with a nominal viscosity at 77°F of 2 cSt, 50 cSt, and 1000 cSt.

EXPERIMENTAL PROCEDURE

Before filling the apparatus, the fluids were degassed by being kept for one hour at 120°F.

Where a free upper surface was used, the supporting bridge for the probe was set up so that the probe was at the approximate center of the test chamber, and in a horizontal position. The probe was lowered into the fluid to a point a few thousandths of an inch above the heated surface, where final adjustments were made to ensure that it was parallel to the surface. It was then moved down until it just touched the heater plate (as indicated by the completion of an electrical circuit). The driving screw was next backed off until any small backlash was taken up, and the dial gage was set to zero. With the sheathed probe (Fig. 6) the procedure was to adjust the micrometer screw until the bottom of the probe was flush with the lower surface of the glass when the reading of the micrometer screw was noted. The glass plate and probe support were placed on the ball bearings and the assembly moved to the point where temperature readings were desired. The probe was then zeroed by the electrical technique previously described.

The temperature measuring probes had junctions of such a size that they could not be located at the lower surface when the probe was zeroed. However, close to the heated surface, the r.m.s. of the temperature fluctuations was proportional to the distance from the surface [27], and this was used to deduce the true location of the measuring junction.

Before each run the power input was set to the chosen level and the system allowed to come to thermal equilibrium, twenty-four hours usually sufficed for this. Once equilibrium was

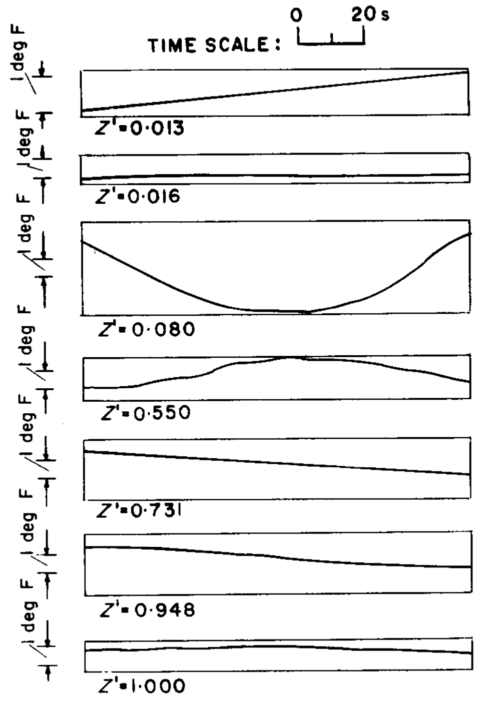

FIG. 8. Sample temperature traces: rigid–rigid (L = 0·551 in., T_H = 137·5°F, T_c = 111·7°F, Ra = 1·31 × 10^5, 50-cSt silicone fluid).

attained, the probe zero was rechecked and adjusted as necessary to allow for any thermal expansion which may have occurred.

The temperature at various stations in the fluid was measured and the e.m.f. output of the probe recorded on the oscillograph (see Figs. 8 and 9) for further analysis. The probe was held at each station for about 5 min. The time average e.m.f. at each position of the probe was determined by averaging the e.m.f. values taken

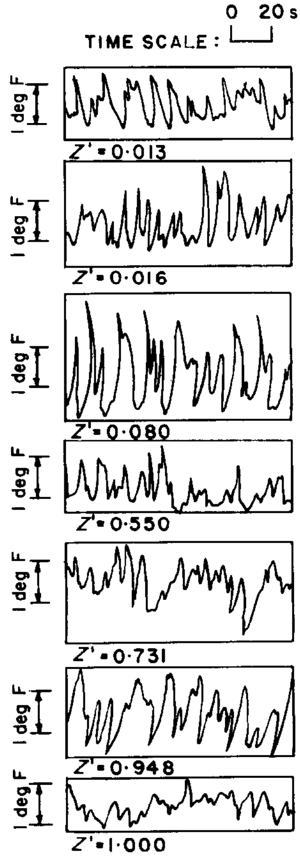

FIG. 9. Sample temperature traces: rigid–rigid (L = 0·551 in., T_H = 119·0°F, T_c = 102·4°F, Ra = 2·03 × 10^6, 2-cSt silicone fluid).

from the trace at 10-s intervals. In a similar manner, the deviations from the mean e.m.f., i.e. the e.m.f. corresponding to the temperature fluctuations, were taken every 10-s and used to calculate the root mean square value of the temperature fluctuations. Immediately before and after each run the temperatures of the upper and lower surfaces of the test chamber were measured, the power input and room temperature were also noted. At the completion of each

Table 1. Experimental conditions

Boundary conditions	Fluid viscosity (cSt)	Temperature (°F) lower surface	upper surface	mean	difference	Prandtl number	Rayleigh number	Regime* (reference [23])
Rigid–rigid	1000	100·6	92·7	96·7	7·9	6920	$1{\cdot}16 \times 10^3$	conduction
	1000	133·3	116·7	125·0	16·6	5210	$3{\cdot}04 \times 10^3$	laminar
	50	110·0	104·0	107·0	6·0	367	$2{\cdot}58 \times 10^4$	transition
	50	124·4	115·4	120·0	9·0	324	$4{\cdot}33 \times 10^4$	transition
	50	137·5	111·7	124·6	25·8	311	$1{\cdot}31 \times 10^5$	turbulent
	2	106·8	101·2	104·0	5·6	17·6	$6{\cdot}71 \times 10^5$	turbulent
	2	119·0	102·4	110·7	16·6	16·3	$2{\cdot}03 \times 10^6$	turbulent
Rigid–free	50	105·4	100·3	102·9	5·1	382	$2{\cdot}18 \times 10^4$	turbulent
	50	132·5	122·0	127·2	10·5	303	$5{\cdot}45 \times 10^4$	turbulent
	2	110·9	107·6	109·3	3·3	16·8	$4{\cdot}32 \times 10^5$	turbulent

* Does not necessarily apply to rigid–free case.
The depth of the fluid layer was 0·551-in for all the experiments.

run in the rigid–rigid case, the probe was moved to a new position in the horizontal plane. Several hours were then allowed to elapse before measurements of the temperature distribution were made at the new coordinate. Ten sets of measurements were made and the experimental conditions are summarized in Table 1.

CONDUCTION, LAMINAR AND TRANSITION REGIMES

Measurements made at a Rayleigh number of $1{\cdot}16 \times 10^3$ showed that the temperature distribution was linear except for $z/L > 0{\cdot}85$, here some slight deviation of the temperature occurred. The hole used to insert the probe through the glass may have introduced a local non-rigid upper boundary condition.

Figure 1 shows that the flow pattern in the laminar regime is predominently in the form of rolls* with their axes parallel to one side of the test cell (the x-axis), although some portions of the fluid exhibit a cellular* flow pattern.

The roll flow pattern has been observed only once before, in the experiments of Chandra [2] and Dassanyake [5] with free convection in a horizontal layer of gas. Sutton [25] has made a careful study of these experiments and has deduced a criterion for the appearance of the more familiar cellular mode of motion. It was found that the conditions of this experiment exceeded the critical conditions of Sutton by only a small amount. This would seem to be in agreement with the mixed flow pattern shown in Fig. 1.

Isothermal plots were made by taking temperature profiles at a number of points along the x- and y-axes. These consisted of "peaks" and "valleys" of temperature, the former corresponding to upward flow of the fluid and the valley to descending fluid flow. This effect was much more pronounced along the y-axis than in the x-direction (axis of the roll) which probably arises from the flow pattern not being perfectly two-dimensional.

The measured isothermals were assumed to represent the flow in a two-dimensional roll with center line located at the longitudinal axis of an upward flowing stream of fluid and the outer edge located at the bottom of a temperature "valley" (center line of descending fluid stream). The temperature profiles obtained from the isothermal plots are shown in Fig. 10. The measured temperature distributions are compared with the values given by Nakagawa's [17]

* Sketches of these two types of flow will be found in reference [1].

theory for a two-dimensional roll. The agreement between theory and experiment is very poor for the ascending fluid column (Fig. 10a) at the center of the roll. It appears that the measured profile is much steeper than predicted by the theory. On the other hand, at the outer edge of the roll (Fig. 10b) the difference between measured and predicted values is much less. Theoretically, all the heat transfer at the point represented by Fig. 10(c) should be by conduction; that such is not the case is clearly seen. In fact, the temperature distribution is very close to that observed at the edge of the roll (Fig. 10b). It is interesting to note that of all possible distributions, the center line distribution (Fig. 10a) most closely resembles the straight line of pure conduction. Perhaps a region of stagnant fluid exists at the base of the ascending column. The deviation from true two-dimensional flow, as represented by the waviness in plan view of the flow pattern (Fig. 1), could be a cause of the lack of agreement between theory and experiment.

The measurements make clear the reason for the good agreement that is found [10, 24] to exist between Nakagawa's theory and Silveston's experimental results. Most of the flow field (in the rolls at least) has a temperature distribution close to that observed in the descending column of fluid, where the agreement between the measured and predicted distribution is best.

In the transition regime, the flow pattern is very irregular (see Fig. 2) and appears to consist of "hexagon-like" figures and extended hexagons. There is a little unsteadiness in the flow pattern, but it seems to have no effect on the temperature traces. The temperature distributions are very similar in both high and low Rayleigh-number cases. This might be anticipated from Silverton's measurements [23], where for a Rayleigh number of about 3×10^4, the Nusselt number approaches a condition of no dependence on the Rayleigh number. Some typical data points for a Rayleigh number of $2{\cdot}58 \times 10^4$ are shown in Fig. 11. Because of the

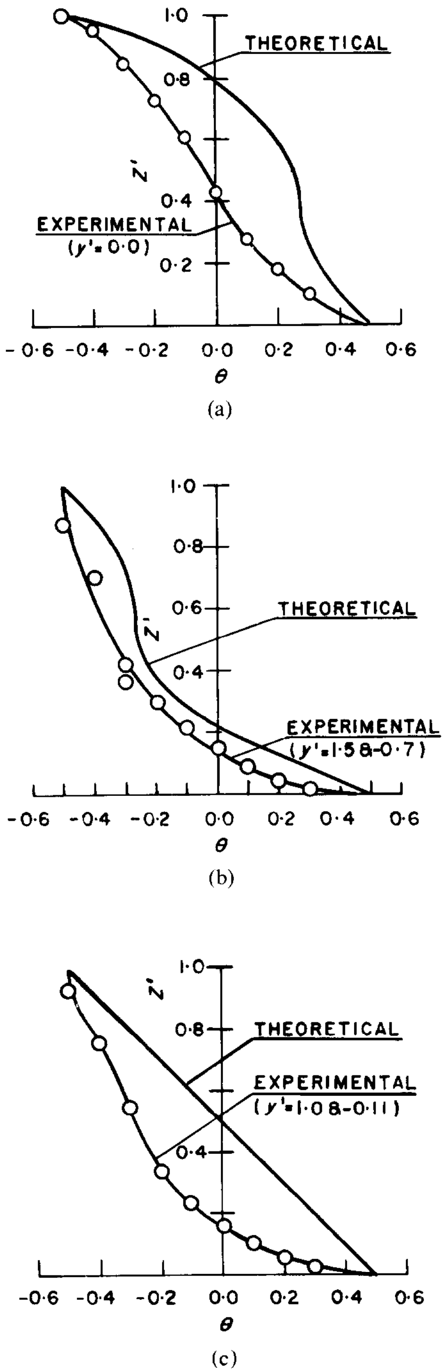

FIG. 10. Temperature distribution, $Ra = 3{\cdot}04 \times 10^3$.
(a) Ascending current.
(b) Descending current.
(c) Conduction.

irregular nature of the observed flow pattern, the measured temperature profiles differ very little from one another. For completeness, the theoretical [17] temperature distribution in a hexagonal cell are shown in Fig. 11. The theory predicts a reversal of the temperature gradient, which is not observed experimentally. As with the laminar regime, the difference between theory and experiment is greatest where there is an ascending fluid stream, also the effects of conduction appear to be more important than the theory predicts.

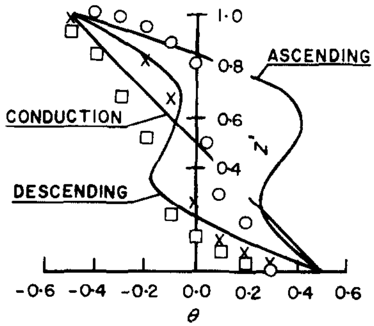

FIG. 11. Temperature distribution, $Ra = 2{\cdot}58 \times 10^4$ (theoretical temperature distribution shown by solid line. ○ Ascending fluid, $x' = 0{\cdot}56$. □ Descending fluid, $x' = 0{\cdot}36$. × Conduction, $x' = 0{\cdot}10$).

TURBULENT REGIME

Visual observation showed that the flow pattern under turbulent conditions was in continual movement, but the speed of movement was less with the more viscous fluids and the rigid upper boundary. Photographs of the observed flow patterns with the 50-cSt silicone fluid are shown in Figs. 3 and 4.

From the temperature traces it can be seen (Fig. 9) that rapid variations of temperature occur in the 2-cSt silicone fluid at points as close to the rigid surface as the probe could approach. Whereas, with the 50-cSt silicone fluid (Fig. 8), the magnitude of the temperature variations is much greater, but the frequency of oscillation is less. This difference in behavior is perhaps more than would be expected from a tenfold difference in Rayleigh number, and might be due to the comparatively high viscosity of the 50-cSt fluid.

The plots (Figs. 12 and 13) of the mean temperature have a "boundary layer" character,

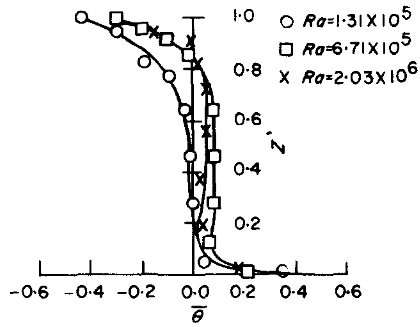

FIG. 12. Mean temperature distribution, rigid–rigid system (some data points omitted for clarity).

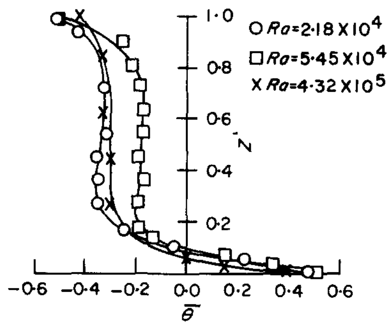

FIG. 13. Mean temperature distribution, rigid–free system (some data points omitted for clarity).

which is especially obvious with the rigid upper surface in place. The existence of a "core" of constant temperature does not agree with the formula proposed by Malkus [14] for the temperature distribution at moderate Rayleigh numbers, viz.,

$$\bar{\theta} \propto \cot(\pi z') \quad \left(z' \gg \frac{1}{Nu - 1}\right). \tag{1}$$

Graphs of $\bar{\theta}$ against $\cot(\pi z')$ for the rigid–rigid case show that the distribution for a Rayleigh number of $1{\cdot}31 \times 10^5$ is in closest agreement with the above formula, the deviation becoming greater as the Rayleigh number increases, as might be expected.

As Thomas and Townsend [26] observed in their experiments (Rayleigh number of $6{\cdot}75 \times 10^5$), the temperature profile in the rigid–rigid case has a region where the temperature gradient reverses. Although they attributed this to the effect of large eddies of the same scale as the apparatus, motions of this type were not observed in these experiments.

The measured temperature distributions are seen to be asymmetrical, as were those of Thomas and Townsend. This may be caused by a non-uniform temperature distribution at the upper surface. Nothing definite is known about conditions in the rigid–free case because the upper surface temperature was obtained, at one point, from the probe. The two thermocouples in the glass plate did not differ by more than 1 degF, however the general circumstances of cooling (by free convection or by a stream of air) and the low conductivity of the glass suggest that large temperature differences might exist between different points on the upper surface.

By graphically measuring the gradient of the temperature profiles it was possible to show the variation with height of the ratio (R) of convective heat transport to the total heat transport (Figs. 14 and 15). From these curves it can be seen that the asymmetric profile

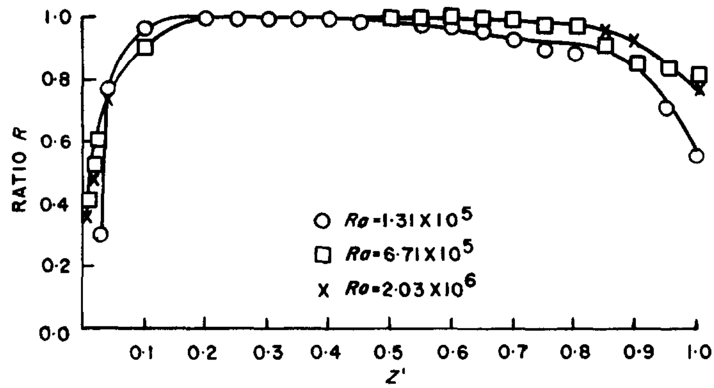

FIG. 14. Distribution of ratio of convective to total heat transport in rigid–rigid system.

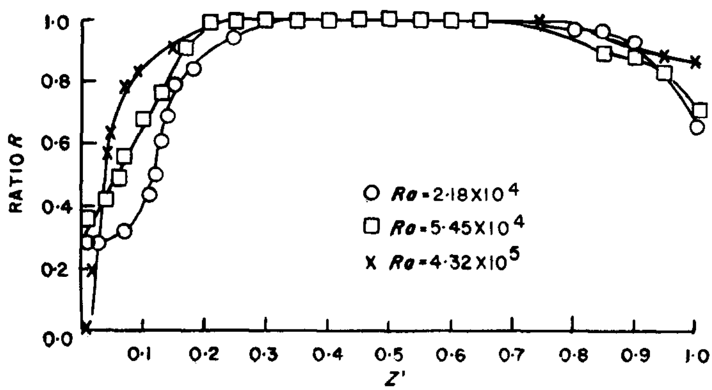

FIG. 15. Distribution of convective to total heat transport in rigid–free system.

results in convection dominating conduction at the upper surface.

Kraichnan [12] has shown that at a height of $z_\alpha(\sim L/2Nu)$ above the lower surface half the heat is transported by conduction. Malkus [14] has given a general result of the same type. These values of z_α/L are shown in Table 2. Since the values of z_α/L obtained from Figs. 14 and 15 are considered to be accurate to about 2 per cent with R of the order 0·5, then the difference between the measured and calculated values must be due to a significant effect.

Kraichnan [12] showed that the temperature distribution would be correlated by an expression of the form

$$\bar{\theta} = A(z/z_\alpha)^{-n} \tag{2}$$

where n depends on z/z_α and has the values given in Table 3. The experimental values of $\bar{\theta}$ and z/z_α were plotted* on bilogarithmic coordinates (see Figs. 16 and 17) and values of the exponent n obtained from these plots are shown in Table 3. For the rigid–rigid lower case there are two temperature regions, one close to the heated surface where $n = 1$ and one further from the sur-

* It was found that a better correlation was obtained if Kraichnan's value of z_α was used rather than the measured value.

Table 2. Values of z_α/L

Boundary condition	Rayleigh number	Nusselt number	z_α/L		
			Kraichnan	Malkus	measured (Figs. 14 and 15)
Rigid–rigid	$1{\cdot}31 \times 10^5$	4·5	0·11	0·10	0·030
	$6{\cdot}71 \times 10^5$	7·3	0·069	0·06	0·0125
	$2{\cdot}03 \times 10^6$	8·9	0·056	0·05	0·0125
Rigid–free	$2{\cdot}18 \times 10^4$	4·1	0·123	0·11	0·12
	$5{\cdot}45 \times 10^4$	5·5	0·091	0·082	0·055
	$4{\cdot}32 \times 10^5$	6·4	0·078	0·070	0·032

Table 3. Values of exponent "n" in equation (2)

Boundary condition	Region	Mean temperature		Temperature fluctuations* ("possible" values)	
		$z < z_c$	$z > z_c$	$z < z_c$	$z > z_c$
Rigid–rigid	lower	1	$\frac{1}{3}$	1	—
	upper	—	$\frac{1}{3}$	no effect observed	
Rigid–free	lower	—	$\frac{1}{3}$–$\frac{1}{2}$	1	—
	upper	—	$\frac{1}{3}$	—	—
Kraichnan		1	$\frac{1}{3}$	1	$\frac{1}{3}$

* Obtained by approximating the integral over z by a finite difference sum.
Note: z_c represents the height at which the value of the exponent n changes; its value for $\bar{\theta}$ can be deduced from an inspection of Figs. 16 and 17 and for $\bar{\theta}'$ from similar plots. In Kraichan's theory $z_c = z_v$.

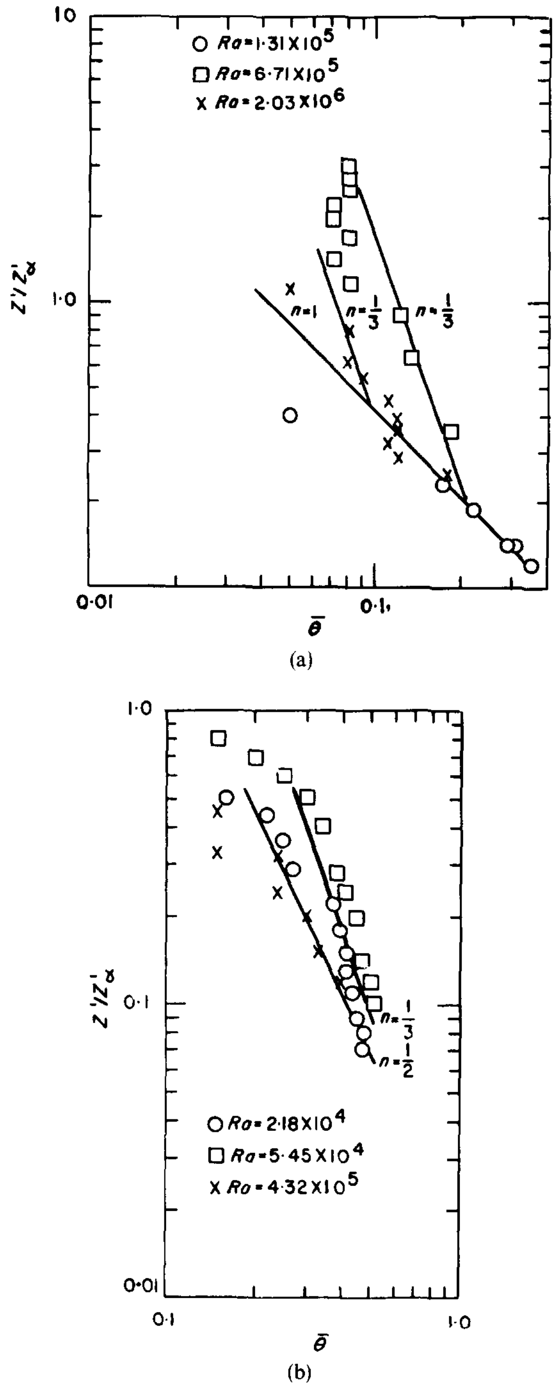

FIG. 16. Kraichnan's correlation of the mean temperature distribution, lower surface.
(a) Rigid–rigid system.
(b) Rigid–free system.

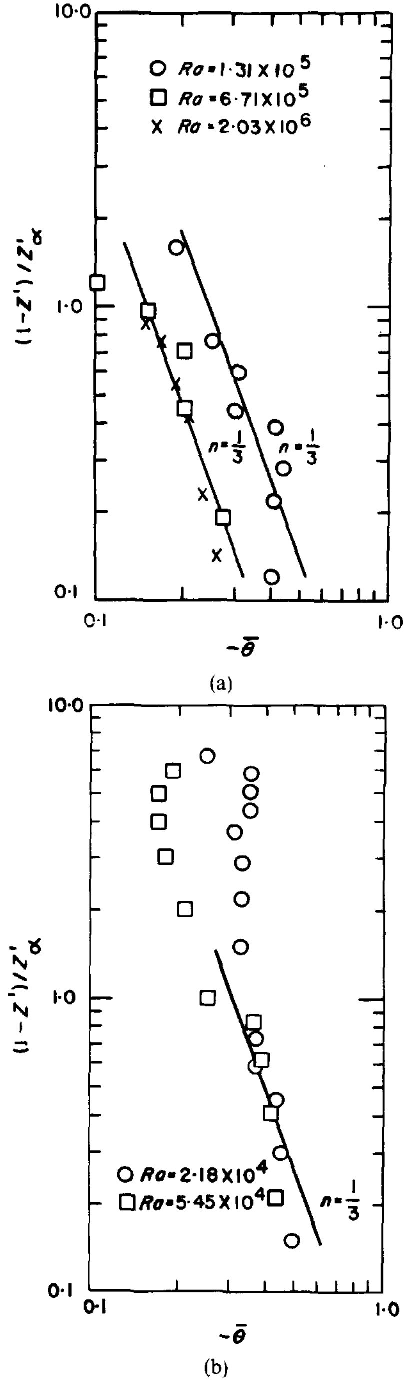

FIG. 17. Kraichnan's correlation of the mean temperature distribution, upper surface.
(a) Rigid–rigid system.
(b) Rigid–free system.

face where $n = \frac{1}{3}$. In Fig. 16 the run with a Rayleigh number of $4{\cdot}32 \times 10^5$ is better represented by $n = \frac{1}{2}$. According to Kraichnan, the exponent is unity for heights between z_α and z_v and $\frac{1}{3}$ for heights greater than z_v. It can be seen that the measured exponents do not apply in exactly the region of the flow field predicted by Kraichnan. However, if the measured values of z_α are used, the situation can be improved. The agreement (Table 4) is generally poor between the experimental and theoretical values of the constant (A) in the equation (2). Kraichnan advances no strong claims for the quantitative aspects of his theory, so it is unreasonable to expect great accuracy in the constant. The discrepancies between theory and experiment may be due to the comparatively high Prandtl number of the fluids, which is not in agreement with Kraichnan's assumption of a Prandtl number of about 0·3. Furthermore, both Kraichnan and Malkus imply that the boundary conditions are free. The correlation is not perfect, because some residual Prandtl number effect is shown by the separation of the lines for different fluids, and in some cases for different Rayleigh numbers.

Figures 18 and 19 show the data plotted using the dimensionless quantities proposed by Howard [11]. The correlation is particularly successful for the temperature distribution close to the heated surface ($\xi \leqslant 2$). However, further from the lower surface the effects of differing Prandtl numbers, which is not taken into account, and, probably the different mean temperature (T_M), manifest themselves. For the upper surface of the rigid–rigid case there was no confirmation of Howard's theory. Considering that Howard's theory only provides an upper bound to the solution, the agreement between theory and experiment is satisfactory.

The distribution of temperature fluctuations has a characteristic form, as can be seen in Figs. 20–22. The maximum value appears (Table 5) to occur at a height above the lower surface coinciding with the height (z_α) at which Kraichnan predicts the heat transfer by turbulent effects will predominate over the heat transfer by molecular effects. This point is also where it was shown experimentally (see Figs. 14 and 15) that about 80 per cent of the heat transfer is occurring by convection, thus it may be said that the peak in the temperature fluctuations is approximately at the outer edge of the "boundary layer". The fluctuations show no agreement with Howard's theoretical curve. It is also apparent that there is no correlation of the temperature fluctuations using Howard's method.

In Table 3 are shown exponents (obtained from plots of $\tilde{\theta}'/\Delta T$ against z/z_α on bilogarithmic coordinates) for the power law representation of the temperature fluctuations which, according to Kraichnan, should have the same form as

Table 4. Values of constant "A" in equation (2)

Boundary conditions	Region	Mean temperature		Temperature fluctuations	
		$z < z_c$	$z > z_c$	$z < z_c$	$z > z_c$
Rigid–rigid	lower	0·042	0·074–0·12	0·062–0·145	—
	upper	—	0·15–0·30	—	—
Rigid–free	lower	—	0·14–0·22	0·053	—
	upper	—	0·14–0·31	—	—
Kraichnan		0·17	0·18	0·17	0·18

Note: z_c represents the height at which the value of the exponent n changes; its value for $\bar{\theta}$ can be deduced from an inspection of Figs. 16 and 17 and for $\tilde{\theta}'$ from similar plots. In Kraichnan's theory $z_c = z_v$.

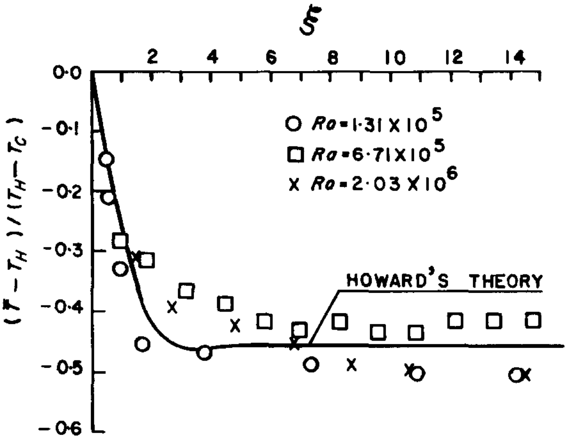

FIG. 18. Howard's correlation of the mean temperature distribution: rigid–rigid system, lower surface.

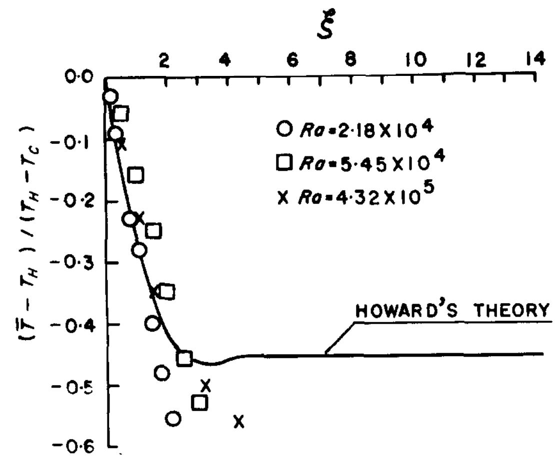

FIG. 19. Howard's correlation of the mean temperature distribution: rigid–free system, lower surface.

Table 5. Location of the maximum of r.m.s. temperature fluctuations

Boundary condition	Rayleigh number	z/z_α	
		lower	upper
Rigid–rigid	$1{\cdot}31 \times 10^5$	1·17	1·64
	$6{\cdot}71 \times 10^5$	1·02	0·20
	$2{\cdot}03 \times 10^6$	1·00	1·16
Rigid–free	$5{\cdot}45 \times 10^4$	1·19	0·55
	$4{\cdot}32 \times 10^5$	0·85	1·28

equation (2). The experiments show, however, that the temperature fluctuations and mean temperature are not necessarily represented by the same formula for the same value of z. Thus, for the temperature fluctuations the exponent in equation (2) has a value unity for z greater than z_α but, as has been pointed out earlier, z is less than z_α when this is true for the mean temperature.

The significantly lower level of turbulent fluctuations in this experiment compared to that predicted by theory is also reflected in the values of the mean square temperature fluctuations $(\overline{\theta'^2})$; as shown in Table 6. The great difference

Table 6. Mean square temperature fluctuations

Boundary conditions	Rayleigh number	$\overline{\theta'^2}$	
		ref [14]	experimental*
Rigid–rigid	$1{\cdot}31 \times 10^5$	15·0	0·81
	$6{\cdot}71 \times 10^5$	0·66	0·05
	$2{\cdot}03 \times 10^6$	4·80	0·20

* Obtained by approximating the integral over z by a finite difference sum.

between the theoretical and experimental results is in agreement with earlier observations [26]. The response of the probe may be such that errors in the measurement of fluctuations would arise from unrecorded high frequency oscillations. The comparatively crude technique used

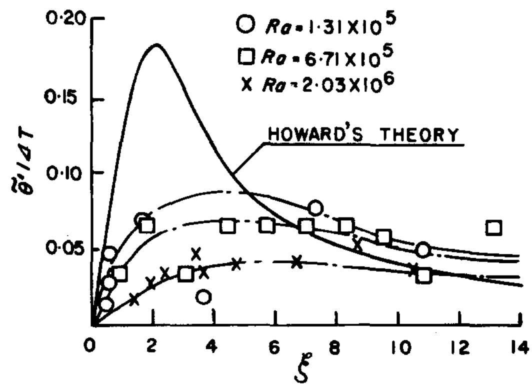

FIG. 20. Root-mean-square temperature fluctuations, rigid–rigid system, lower surface.

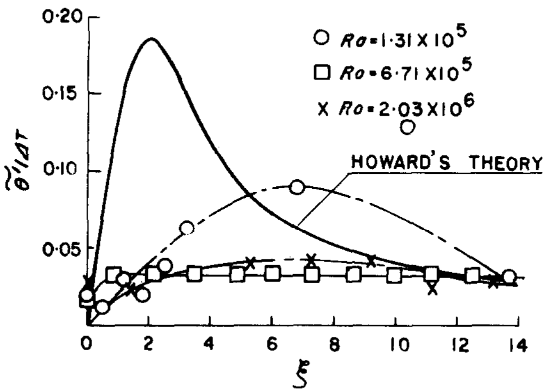

FIG. 21. Root-mean-square temperature fluctuations, rigid–rigid system, upper surface.

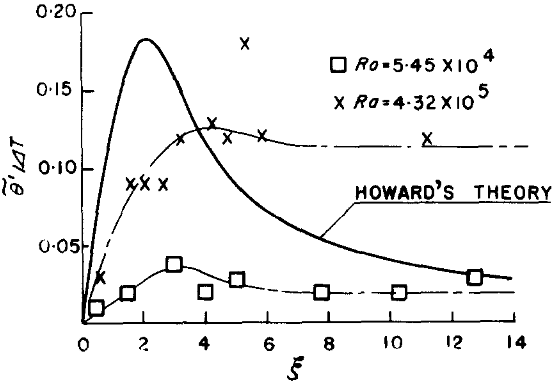

FIG. 22. Root-mean-square temperature fluctuations, rigid–free system, lower surface.

for determining the mean square temperature fluctuations may not provide sufficiently accurate figures to compare with the theoretical calculations.

ERROR CONSIDERATIONS

The measured values of θ or $\bar{\theta}$ are estimated to have an uncertainty varying from 4·3 per cent for the largest temperature difference between the hot and cold surfaces, to 8·3 per cent where the temperature difference is smallest. The dimensionless r.m.s. temperature fluctuations ($\tilde{\theta}'/\Delta T$) are considered to be accurate to 15 per cent. The value of the dimensionless height (z/L) has an uncertainty varying from 0·3 to 25 per cent, depending on how close the measurements are made to the rigid boundaries. Assuming that there is no deviation in the height measurement, all the data has a deviation from the plotted mean which is less than the estimated uncertainty of the data. Some of the measurements were carried out twice and the repeatability was found to be within twice the minimum uncertainty for θ and $\bar{\theta}$ and within the estimated uncertainty for $\tilde{\theta}'/\Delta T$.

CONCLUSIONS

The fluid flow in the laminar regime was found to be in the form of rolls and square planform cells. This is in agreement with Sutton's [25] criterion for the appearance of this mode of motion. The temperature measurements did not compare well with the theory [17] because of the high Rayleigh number. In the transition regime a cellular mode of flow was observed but, as expected, the difference between theoretical and measured temperature distributions was considerable.

For high Rayleigh numbers the mean temperature distribution can be expressed by a relation of the form

$$\bar{\theta} = A(z/z_\alpha)^{-\frac{1}{3}} \tag{3}$$

as was shown by Kraichnan's [12] calculations. However, this formula did not appear to apply in exactly the region of the flow field predicted by Kraichnan. A study of the heat transport mechanism showed that the region where conduction predominates is considerably thinner than expected [12, 14]. The reason for these discrepancies may be because of the comparatively low Prandtl number assumed in reference [12] or the boundary conditions [12, 14]. According to Kraichnan there should be a region near the heated surface where the temperature is inversely proportional to the distance from the surface; only one such case was found (in a high-Prandtl-number fluid). Close to the heated surface the measurements were in good agreement with Howard's [11] theoretical results. The level of the r.m.s. temperature fluctuations was much less than the theoretical values [11, 14], and this may be due to the method of measurement.

This investigation shows that further studies are required of the conditions in a heated fluid layer. It is planned to carry out experiments in the turbulent regime at high Prandtl and Rayleigh numbers, and the results will be reported in due course.

ACKNOWLEDGEMENTS

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Résumé—Une étude expérimentale de la distribution de température et de l'écoulement dans une couche horizontale de liquide chauffée par en-dessous a été effectuée. Les expériences couvraient une gamme de nombres de Rayleigh allant de $1{,}16.10^3$ à $2{,}03.10^6$. La surface supérieure du liquide était, dans certains cas, libre et, dans d'autres cas, en contact avec une plaque de verre. Des silicones liquides de viscosité 2, 50 et

1000 cSt ont été utilisés. Les configurations de l'écoulement ont été observées en mélangeant aux fluides une petite quantité de poussière d'aluminium.

L'écoulement en régime laminaire consiste en une combinaison de "rouleaux" et de cellules de forme carrée. Les mesures de températures n'étaient pas exactement en accord avec les prévisions théoriques.

Dans le régime de transition, la configuration de l'écoulement était cellulaire. Les mesures de température ont été comparées avec la théorie et l'on a trouvé une différence considérable.

Pour des nombres de Rayleigh élevés, la distribution de la moyenne temporelle de la température au voisinage des frontières était inversement proportionnelle à la distance à la surface. L'accord avec la théorie était tout-à-fait bon. La nature des processus de transport de chaleur a été également étudiée. Le niveau de la moyenne quadratique des fluctuations de température était beaucoup plus faible que les valeurs théoriques.

Zusammenfassung—Die Temperaturverteilung und die Strömung in einer waagrechten, von unten beheizten Flüssigkeitsschicht wurde experimentell untersucht. Die Versuche umfassten einen Rayleigh zahlenbereich von $1{,}16 \times 10^3$ bis $2{,}03 \times 10^6$. Die obere Begrenzung der Flüssigkeit war in einigen Fällen zur Umgebung geöffnet, in anderen Fällen von einer Glasplatte gebildet. Silikonflüssigkeiten mit Zähigkeiten von 2, 50 und 1000 cSt wurden verwendet. Die Strömungsmuster wurden durch Beimischen kleiner Mengen von Aluminiumpulver zur Flüssigkeit sichtbar gemacht.

Die Strömung im Laminarbereich bestand aus einer Kombination von "Rollen" und im Aufriss quadratischen Zellen. Die Temperaturmessungen stimmten nicht eng mit theoretischen Vorhersagen überein. Im Übergangsgebiet war das Strömungsmuster zellular. Die Temperaturmessungen wurden mit der Theorie verglichen wobei sich eine erhebliche Abweichung ergab. Bei hohen Rayleighzahlen war die zeitlich mittlere Temperaturverteilung nahe den Begrenzungen umgekehrt proportional zur Kubikwurzel des Abstands zur Begrenzung. In einem Fall (einer Flüssigkeit mit grosser Prandtlzahl) fand sich ein zusätzlicher Bereich nahe der Oberfläche, wo die Temperatur umgekehrt proportional zum Abstand von der Oberfläche war. Die Übereinstimmung mit der Theorie war ziemlich gut. Die Natur des Wärmetransports wurde ebenfalls untersucht. Die Höhe der Temperaturschwankungen nach der Wurzel der mittleren Quadrate war viel geringer als die theoretischen Werte.

Аннотация—Проведено экспериментальное исследование распределения температур и характера течения в горизонтальном слое нагреваемой снизу жидкости. Эксперименты проводились в диапазоне значения критерия Релея от $1{,}16 \times 10^3$ до $2{,}03 \times 10^6$ В некоторых случаях верхняя поверхность жидкости была открытой, а в некоторых соприкасалась со стеклянной пластинкой. Использовались кремний-органические жидкости с вязкостью 0,025 и 10 стокс. Наблюдения за картиной течения проводились с помощью примесей небольшого количества алюминиевой пыли.

При ламинарном режиме поток жидкости представлял собой комбинацию «круглых валов» и квадратных плоских ячеек. Измеренные значения температур не совпадают точно с расчетными.

При переходном режиме течение имело ячеистый характер. Измеренные значения температур сравнивались с теоретическими, и обнаружено значительное расходение.

При больших значениях числа Релея распределение усредненных во времени температур вблизи границ обратно пропорционально кубическому корню расстояния от границы. В одном случае (жидкость с большим значением числа Прандтля) вблизи поверхности сущестовала дополнительная область, где температура была обратно пропорциональна расстоянию от поверхности. Согласование с теорией было довольно хорошим. Проводились также исследования характера процессов переноса тепла. Уровень среднеквадратичных температурных пульсаций был намного ниже теоретических значений.